Estimating Earnings Risk

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Macroeconomics III

Overview

- The aim is to study the consequences of idiosyncratic earnings risk.
- We will start with a simple model where risk is purely exogenous to me.
- We start with an econometric model for earnings:

$$ln(E_{ih}) = e_{ih} = \beta X_{ih} + u_{ih}.$$

- Log earnings can be decomposed into:

  a deterministic (observable) component (βX_{ih}).

  a stochastic (unobservable) component (u_{ih}).

  implies shocks are proportional to log earnings.

The Stochastic Component

- An early approach (see MaCurdy (1982)) assumes that there are persistent and transitory shocks following:

$$u_{ih} = \alpha_i + z_{ih} + \tau_{ih}$$
$$\tau_{ih} = MA(q)\iota_{ih}$$
$$z_{ih} = \rho z_{ih-1} + \epsilon_{ih}$$

- ι_{ih} are transitory shocks including:

  bonuses.

  short sickness.

  strikes.

  inflation.

- α_i is permanent heterogeneity.

The Stochastic Component

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$$\tau_{ih} = MA(q)\iota_{ih}$$
$$z_{ih} = \rho z_{ih-1} + \epsilon_{ih}$$

- ϵ_{ih} are persistent shocks including:

  shifts in idiosyncratic labor demand.

  promotions.

  job-ladder effects.

  losing a high tenured job.

- Usually these models are estimated by General Methods of Moments.

General Methods of Moments (*GMM*)

- Suppose our model creates a total of $\mathcal{M}(p)$ moments.
- We choose a subset $M(p)$ for estimation. Example:

  Output is 3 in quarter 1, 2 in quarter 2, and 2.5 in quarter 3...

  Our moments may be the mean, standard deviation, autocorrelation of output.

- Let $\tilde{p}$ be the true parameters, and $\hat{M}$ be the sample analogous to M. If our model is correct:

$$\mathbb{E}(\hat{M}(\tilde{p}) - M(\tilde{p})) = 0.$$

GMM Estimation

- Assume you know the moment generating function for some parameters p: $g(X_t, p)$.

- GMM performs

$$p = \underset{p}{argmin}((g(X_t, p) - \hat{M}(p))W(g(X_t, p) - \hat{M}(p))')$$

  W is an appropriate (positive-definite) weighting matrix.

- When number of moments equal to number of parameters, we have exact identification: MM.

  Having more moments increases efficiency.

  Allows us to test our overidentified model.

Weighting Matrix

- Often, studies use the identity weighting matrix.

Weighting Matrix

- Often, studies use the identity weighting matrix.
- Intuitively, we would like to give more weight to moments estimated with high precision. Thus, use the variance-covariance structure.

$$p = \underset{p}{argmin}((g(X_t, p) - \hat{M}(p))W(g(X_t, p) - \hat{M}(p)))'$$

Is asymptotically normally distributed with variance:

$$V = (D'WD)^{-1}D'WSWD(D'WD)^{-1}$$

$$S = \mathbb{E}\Big(\sum_{t=1}^{T}(g(X_t, p) - \hat{M}(p))^2\Big)$$

$$D = \mathbb{E}\Big(\sum_{t=1}^{T}\frac{\partial(g(X_t, p) - \hat{M}(p))}{\partial p}\Big)$$

Weighting Matrix II

Optimal weighting matrix: $W = S^{-1}$

$$V = (D'WD)^{-1}D'WSWD(D'WD)^{-1} = (D'S^{-1}D)^{-1}$$

- Preferred moment conditions have small S and large D.
- Moments have small sample variation.
- Moments are informative on p.

Feasible GMM

In general, S depends on the parameter vector p.

1. Start with an initial weighting matrix (e.g., identity matrix).
2. Estimate $\hat{S}$.
3. Use $W = \hat{S}$.
4. Iterative GMM: Continue procedure until convergence.

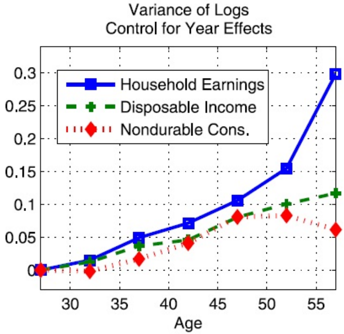
GMM Estimation of Earnings Risk

Which Moments? Two Approaches

Micro approach: Estimate by GMM on covariance matrix earnings growth,

$$g_{ih} = u_{ih} - u_{ih-1}.$$

Macro approach: Estimate by GMM on covariance matrix of life-cycle variance, $Var(u_{ih})$:



Obtaining Residuals

$$e_{ih} = \beta X_{ih} + u_{ih}.$$
$$G_{ih} = \Delta\beta X_{ih} + \Delta u_{ih}.$$

- The first step is to obtain residuals.
- Macro approach (u_{ih})

  Run by age OLS regressions: $e_{ih} = \beta X_{ih} + u_{ih}$.
- Micro approach (g_{ih})

  Run by age OLS regressions: $G_{ih} = \Delta\beta X_{ih} + \Delta u_{ih}$.
- Assume any deviations from these predictable patters are shocks.

  We as econometricians have the same information set as the individual.

Micro Approach

Residual earnings growth for $\tau_{ih} = \iota_{ih} + \theta\iota_{ih-1}$ and iid shocks:

$$g_{ih} = u_{ih} - u_{ih-1} = (\rho - 1)z_{ih-1} + \epsilon_{ih} + \iota_{ih} + \iota_{ih-1}[\theta - 1] - \theta\iota_{ih-2}$$

We observe data for ages $h = [1, ..., H]$. However, at early ages, the moments depend on unobserved data, i.e., $z_{i0}, \iota_{i0}, \iota_{i-1}$. As we have no observations on these, we have to make some assumption. We will assume

$$z_{i0} \sim N\left(0, \frac{\sigma_\epsilon^2}{1-\rho^2}\right)$$
$$\iota_{i0} \sim N(0, \sigma_\iota^2)$$
$$\iota_{i-1} \sim N(0, \sigma_\iota^2)$$

Moment conditions

This leads to the following variance-covariance function:

$$Var(g_{ih}) = \sigma_\epsilon^2 + (\rho - 1)^2 \frac{\sigma_\epsilon^2}{1-\rho^2} + \sigma_\iota^2[1 + (\theta - 1)^2 + \theta^2]$$

$$Cov(g_{ih}, g_{ih-1}) = (\rho - 1)\sigma_\epsilon^2 + \rho(\rho - 1)^2 \frac{\sigma_\epsilon^2}{1-\rho^2} + \sigma_\iota^2[(\theta - 1)(1 - \theta)]$$

$$Cov(g_{ih}, g_{ih-2}) = \rho(\rho - 1)\sigma_\epsilon^2 + \rho^2(\rho - 1)^2 \frac{\sigma_\epsilon^2}{1-\rho^2} - \theta\sigma_\iota^2$$

$$Cov(g_{ih}, g_{ih-n}) = \rho^{n-1}(\rho - 1)\sigma_\epsilon^2 + \rho^n(\rho - 1)^2 \frac{\sigma_\epsilon^2}{1-\rho^2} \ \forall n > 2$$

Moment Estimation

The variance-covariance matrix (COV) has $\frac{H(H+1)}{2}$ unique moments. Let

$$\hat{M} = vech(COV)$$

Estimation based on

$$p = \underset{p}{argmin}(M(p) - \hat{M})W(M(p) - \hat{M}))',$$

where $p = [\sigma_\epsilon^2 \quad \rho \quad \sigma_\iota^2 \quad \theta]$.

Identification

$$Cov(g_{ih}, g_{ih-n}) = \rho^{n-1}(\rho - 1)\sigma_\epsilon^2 + \rho^n(\rho - 1)^2 \frac{\sigma_\epsilon^2}{1 - \rho^2} \ \forall n > 2$$

Distant lags of earnings growth are related because of mean reversion of persistent shocks. This covariance should be negative for $\rho < 1$, or zero for a random walk.

$$Cov(g_{ih}, g_{ih-1}) = (\rho - 1)\sigma_\epsilon^2 + \rho(\rho - 1)^2 \frac{\sigma_\epsilon^2}{1 - \rho^2} + \sigma_\iota^2[(\theta - 1)(1 - \theta)]$$

If early lags of earnings growth are related beyond the effect of persistent shocks, it indicates there are transitory shocks which are mean-reverting.

$$Cov(g_{ih}, g_{ih-2}) = \rho(\rho - 1)\sigma_\epsilon^2 + \rho^2(\rho - 1)^2 \frac{\sigma_\epsilon^2}{1 - \rho^2} - \theta\sigma_\iota^2$$

The second lag tells us whether this mean reversion takes longer than 1 period.

Macro Approach

Residual earnings for $\tau_{ih} = \iota_{ih} + \theta\iota_{ih-1}$ and iid shocks:

$$u_{ih} = \alpha_i + z_{ih} + \iota_{ih} + \theta\iota_{ih-1}$$

Model is identified by the variance-covariance matrix (assuming $z_{i0} = \epsilon_{i0}$ and $\tau_{i0} = \iota_{i0}$):

$$Var(u_{ih}) = \sigma_\alpha^2 + \sigma_\iota^2[1 + \theta^2] + \sigma_\epsilon^2 \sum_{j=0}^{h} \rho^{2j}$$

$$Cov(u_{ih}, u_{ih-1}) = \sigma_\alpha^2 + \sigma_\iota^2\theta + \sigma_\epsilon^2 \sum_{j=0}^{h-1} \rho^{1+2j}$$

$$Cov(u_{ih}, u_{ih-2}) = \sigma_\alpha^2 + \sigma_\epsilon^2 \sum_{j=0}^{h-2} \rho^{2+2j}$$

$$Cov(u_{ih}, u_{ih-n}) = \sigma_\alpha^2 + \sigma_\epsilon^2 \sum_{j=0}^{h-n} \rho^{n+2j} \ \forall n > 2$$

Moment Estimation

The variance-covariance matrix (COV) has $\frac{H(H+1)}{2}$ unique moments. Let

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Estimation based on

$$p = \underset{p}{argmin}(M(p) - \hat{M})W(M(p) - \hat{M}))',$$

$$\text{where } p = [\sigma_\alpha^2 \quad \sigma_\epsilon^2 \quad \rho \quad \sigma_\iota^2 \quad \theta].$$

Identification

$$Var(u_{ih}) = \sigma_\alpha^2 + \sigma_\iota^2[1+\theta^2] + \sigma_\epsilon^2 \sum_{j=0}^{h-1} \rho^{2j}$$

Inequality growth over the life-cycle because of persistent shocks. If the increase is linear,$\rho = 1$, It is concave for $\rho < 1$.

$$Cov(u_{ih}, u_{ih-n}) = \sigma_\alpha^2 + \sigma_\epsilon^2 \sum_{j=0}^{h-n} \rho^{n+2j} \ \forall n > 2$$

Earnings inequality today and at distant lags are related because of permanent heterogeneity, or because of persistent shocks obtained in the past.

$$Cov(u_{ih}, u_{ih-1}) = \sigma_\alpha^2 + \sigma_\iota^2\theta + \sigma_\epsilon^2 \sum_{j=0}^{h-1} \rho^{1+2j}$$

Inequality today and yesterday are related beyond that because of transitory shocks.

Additional Useful Techniques

Bootstrapping

- The asymptotic distribution of the estimator may be unknown.
- Bootstrapping resamples the sample population many times to compute statistics.
- Example: we have M people and measure the mean height.

  Redraw a random sample with length M and recompute mean.

  Repeat N times.

  Take standard deviation of outcomes.
- When draws are i.i.d., this method works well.
- In our case, observations are not i.i.d.

Block-Bootstrapping

Horowitz (2003) provides a block-bootstrapping procedure for processes that can be approximated by a Markov-process.
Example: Panel of $\mathcal{I}$ individuals with $\mathcal{T}$ observations: $\mathcal{I}(\mathcal{T}-1)$ income growth observations.

1. Randomly draw $\mathcal{I}(\mathcal{T}-1)$ observations.
2. Compute income growth for each block.
3. Compute moments of interest.
4. Repeat N times.
5. Take standard deviation over estimates.

Method of Simulated Moments (MSM)

- So far, we assume we know the moment generating function $g(X_t, p)$.

  As in the *Micro* and *Macro* approaches.

- With more complex DGPs, we may not know this function.

- Simulation based methods are a natural extension of *GMM*.

MSM Idea

- Assume you do not know the moment generating function $g(X_t, p)$.
- You can simulate the *ith* moment $g^i(\hat{X}_t, p)$ for a specific draw of observables $\hat{X}_t$.

  Initialize in the ergodic distribution.

  Use a burn in period at simulation which you disregard.

- Repeat this simulation R times for different draws $\hat{X}_t$.

$$M^i(p) = \frac{1}{R}\sum_{r=1}^{R} g^i(\hat{X}_t^r, p).$$

MSM Estimator

$$\hat{p} = \underset{\hat{p}}{argmin}(M(\hat{p}) - \hat{M})W(M(\hat{p}) - \hat{M}))',$$

With $W = S^{-1}$, Duffie and Singleton (1993) show that asymptotic variance of estimator is $(1 + R^{-1})(D'S^{-1}D)^{-1}$.

- Loss of efficiency is small with R sufficiently large.
- Important to have moments which are informative about p.
- Moments should be estimated with little uncertainty.

References

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